Change point detection in Python

Fundamentals of reproducible research and free software (MVA 2022-2023)

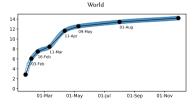
Charles Truong¹

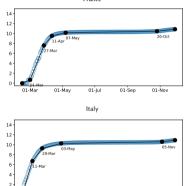
¹Centre Borelli Université Paris-Saclay ENS Paris-Saclay, CNRS

Wednesday 23rd November 2022



- Change point detection is a common task when dealing with non-stationary time series.
- Application example: study of COVID-19 infection curve [Jiang et al., 2020].
- Data from "Our World in Data" (ourworldindata.org).
- Cumulative reported deaths in log-scale.
- Piecewise linear trends (linear spline smoothing with optimal knot selection).
- The slope gives the growth rate ("log-return").





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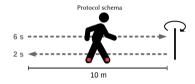
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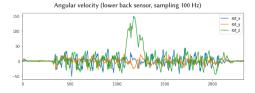
France

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- Application example: automatic diagnosis of neurologically impaired patients [Truong et al., 2019a].

Healthy and pathological subjects underwent a fixed protocol:

- standing still,
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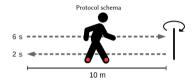


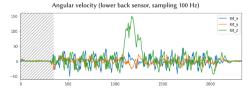


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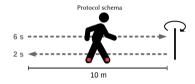


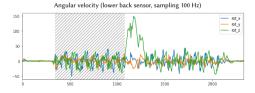


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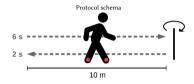


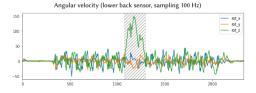


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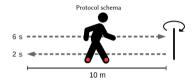


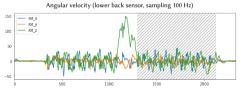


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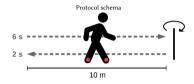


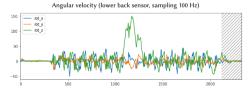


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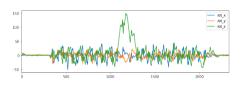
- standing still,
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What is change point detection?

- Change point detection consists in finding the temporal boundaries between homogeneous time periods.
- ▶ Informally: "multivariate signal → list of change point indexes"



Т	z	У	x
	-21.055	8.337	8.139
	-20.693	9.881	6.964
	-19.309	9.993	4.317
	-16.941	8.950	1.752
\rightarrow [t_1, t_2, t_3, \ldots]	-13.143	7.356	-0.305
L 17 27 37 11	-7.361	7.384	-2.320
	-2.530	8.467	-3.312
	2.523	10.891	-3.697
	5.863	13.363	-2.622
L	4.473	11.761	-2.728

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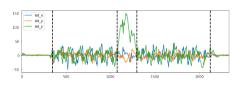


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3. General principle of ruptures

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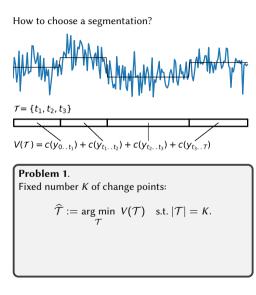
Change in mean and variance (1-D) Change in mean and variance (n-D) Change in distribution (parametric) Change in distribution (non-parametric) Gait analysis

5. Supervised change point detection

General principle Learn the representation

6. Conclusion

General principle



The "best segmentation" is the minimizer, denoted $\widehat{\mathcal{T}}$, of a criterion $V(\mathcal{T})$:

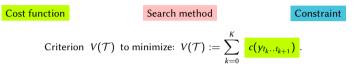
$$V(\mathcal{T}):=\sum_{k=0}^{K} c(y_{t_k\ldots t_{k+1}}).$$

Cost example: $c(y) = \sum_t (y_t - \overline{y})^2$.

Problem 2. Unknown number of change points: $\widehat{\tau} := \underset{T}{\arg \min} V(T) + \operatorname{pen}(T)$ where $\operatorname{pen}(T)$ measures the complexity of a segmentation T.

General principle

Detection methods are the combination of three elements [Truong et al., 2020].



Problem 1. Fixed number K of change points: $\widehat{\mathcal{T}} := \arg \min_{\mathcal{T}} V(\mathcal{T}) \text{ s.t. } |\mathcal{T}| = K$.

Problem 2. Unknown number of change points: $\widehat{\mathcal{T}} := \underset{\mathcal{T}}{\operatorname{arg \,min}} V(\mathcal{T}) + \underset{\mathcal{T}}{\operatorname{pen}(\mathcal{T})}$ where $\operatorname{pen}(\mathcal{T})$ measures the complexity of a segmentation \mathcal{T} .

General principle

A modular architecture.

First import and data loading. [319]: 1 import ruptures as rpt signal = get signal(...) # user defined [3291: 1 # cost function Choosing the cost function 2 c = rpt.costs.CostL2() Here, $c(y) = \sum_{t} (y_t - \overline{y})^2$. Choosing the search method 1 # search method [3301: 2 algo = rpt.Binseg(jump=5, min size=10, custom cost=c) Here, binary segmentation. Fitting the algorithm. [331]: 1 # fit algo 2 algo.fit(signal) Choosing the constraint [332]: 1 # predict change points 2 # fixed number of changes Then detecting the change points ("predict"). 3 bkps = algo.predict(n bkps=10) 4 # or penalized detection 5 bkps = algo.predict(pen=50) Measuring the detection accuracy. 1 from ruptures.metrics import hausdorff [333]: 3 error = hausdorff(true bkps, bkps)

A discrete optimization problem

Minimize the sum of cost over all segmentations:

$$\min_{t_1, t_2, \ldots, t_K} \sum_{k=0}^K c(y_{t_k \ldots t_{k+1}}).$$

or

$$\min_{t_1,t_2,\ldots,t_K}\sum_{k=0}^K c(y_{t_k\ldots t_{k+1}}) + \beta K.$$

- A naive implementation is prohibitive $\binom{T}{K}$ segmentations).
- > The problem is solved recursively using Bellman's dynamic programming.
- For most cost functions, the complexity is $\mathcal{O}(T^2)$ in operations and $\mathcal{O}(T)$.
- Heuristics to approximately solve this problem exist: binary segmentation (with variants) and window-sliding. Complexity in $\mathcal{O}(T)$.

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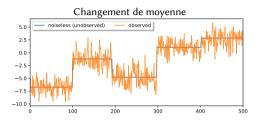
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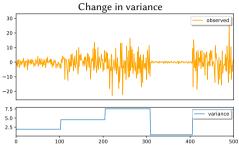
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Change in mean and variance (1-D)





Cost function:

$$c(y_{a..b}) = \sum_{t=a}^{b-1} (y_t - \bar{y}_{a..b})^2$$

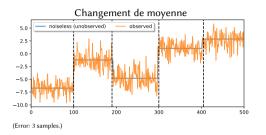
where $\overline{y}_{a..b}$ is the empirical mean of $y_{a..b}$.

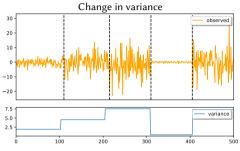
Cost function:

$$c(y_{a..b}) = (b-a)\log(\hat{\sigma}_{a..b})$$

where $\hat{\sigma}_{a..b}$ the empirical standard-deviation $y_{a..b}$.

Change in mean and variance (1-D)





(Error: 10 samples.)

Cost function:

$$c(y_{a..b}) = (b-a)\log(\hat{\sigma}_{a..b})$$

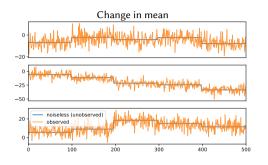
where $\hat{\sigma}_{a..b}$ the empirical standard-deviation $y_{a..b}$.

Cost function:

$$c(y_{a..b}) = \sum_{t=a}^{b-1} (y_t - \bar{y}_{a..b})^2$$

where $\overline{y}_{a..b}$ is the empirical mean of $y_{a..b}$.

Change in mean and variance (n-D)



Change in variance/covariance

Cost function:

$$c(y_{a..b}) = \sum_{t=a}^{b-1} ||y_t - \bar{y}_{a..b}||^2$$

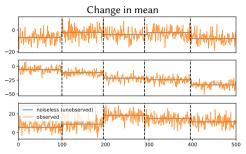
where $\overline{y}_{a..b}$ is the empirical mean of $y_{a..b}$.

Cost function:

$$c(y_{a..b}) = (b-a)\log \det \hat{\Sigma}_{a..b}$$

where $\hat{\sigma}_{a..b}$ is the empirical covariance matrix of $y_{a..b}$.

Change in mean and variance (n-D)

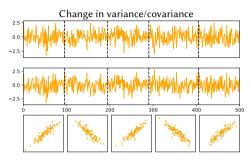


(Error: 7 samples.)

Cost function:

$$c(y_{a..b}) = \sum_{t=a}^{b-1} ||y_t - \bar{y}_{a..b}||^2$$

where $\overline{y}_{a..b}$ is the empirical mean of $y_{a..b}$.



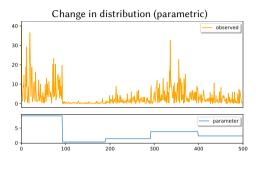
(Error: 3 samples.) Cost function:

$$c(y_{a..b}) = (b-a) \log \det \hat{\Sigma}_{a..b}$$

where $\hat{\sigma}_{a..b}$ is the empirical covariance matrix of $y_{a..b}$.

Change in distribution (parametric)

▶ ...



Cost function:

$$c(y_{a..b}) = -\max_{\theta} \log f_{\theta}(y_{a..b})$$

where f_{θ} is the density of the chosen distribution, parametrized by θ .

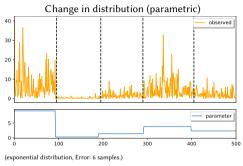
Important fact. The estimated change points converge to the true changes.

[Lavielle, 1999, Detection of multiples changes in a sequence of dependant variables. Stochastic Processes and Their Applications, 83(1), 79–102.]

- Not necessarily i.i.d. observations.
- Can be strongly dependent (but stationary).

Change in distribution (parametric)

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- Not necessarily i.i.d. observations.
- Can be strongly dependent (but stationary).

Change in distribution (non-parametric)

When the underlying distribution is unknown:

- [Arlot et al., 2019, A kernel multiple change-point algorithm via model selection. Journal of Machine Learning Research, 20(162), 1–56.]
- [Matteson and James, 2014, A nonparametric approach for multiple change point analysis of multivariate data. Journal of the American Statistical Association, 109(505), 334–345.]
- [Ross and Adams, 2012, Two nonparametric control charts for detecting arbitrary distribution changes. Journal of Quality Technology, 44(2), 102–117.]

The kernel approach is particularly interesting because it can deal with non-numerical data: symbolic signals, texts, functional time series,...

General principle:

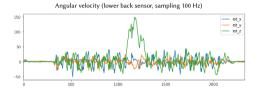
- The signal is mapped to a high-dimensional space: $y_t \longrightarrow \phi(y_t)$.
- Detection of change in the mean of the $\phi(y_t)$. Cost function:

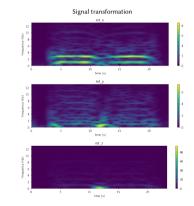
$$c(y_{a..b}) = \sum_{t=a}^{b-1} \|\phi(y_t) - \bar{\mu}\|_{\mathcal{H}}^2$$

where $\bar{\mu}$ is the empirical mean of $\{\phi(y_t)\}_{a..b.}$ (Computed using the kernel trick.)

Gait analysis

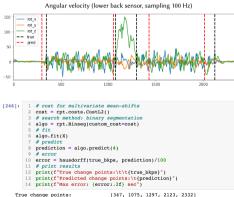
- > To simplify the detection task, the signal is transformed (here, short-term Fourier transform).
- ▶ Then mean-shifts are detected.





Gait analysis

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- Then mean-shifts are detected.



True change points: [347, 1075, 1297, 2123, 2332] Predicted change points: [300, 1055, 1430, 2015, 2332] Max error: 1.33 sec

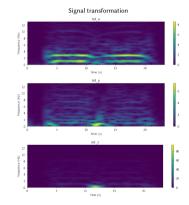


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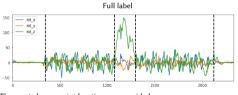
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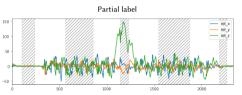
6. Conclusion

Supervised change point detection

- How to integrate expert knowledge to calibrate the change point detection? [Truong et al., 2019b]
- The expert provides the target segmentation: either full or partial label.
- Labels are hard to collect. The easier for the clinicians, the better.



The exact change point locations are provided.



Only homogeneous periods (hatched areas) are provided (weakly supervised).

Labels are transformed into constraints. Intuitively, the problem is:

Learn a transformation Ψ such that $d(\Psi(x_t), \Psi(x_s)) \leq u$ if x_t and x_s similar $d(\Psi(x_t), \Psi(x_s)) \geq l$ if x_t and x_s dissimilar

(u > 0 and l > 0)

Two samples are *similar* if they belong to the same regime.

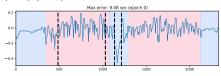
Two samples are *dissimilar* if they belong to consecutive regimes.

This setting can be used to learn a deep representation.

Here, two layers of temporal separable convolutions and maxpooling (with tensorflow).



Epoch by epoch (epoch 0)



True segmentation: alternating colors.

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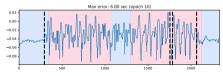
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Epoch by epoch (epoch 10)



True segmentation: alternating colors. Predicted segmentation: dashed lines.

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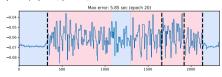
Two samples are *dissimilar* if they belong to consecutive regimes.

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Epoch by epoch (epoch 20)



True segmentation: alternating colors. Predicted segmentation: dashed lines.

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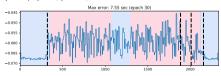
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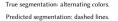
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Here, two layers of temporal separable convolutions and maxpooling (with tensorflow).



Epoch by epoch (epoch 30)





Labels are transformed into constraints. Intuitively, the problem is:

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Two samples are *similar* if they belong to the same regime.

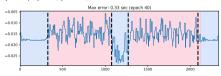
Two samples are *dissimilar* if they belong to consecutive regimes.

This setting can be used to learn a deep representation.

Here, two layers of temporal separable convolutions and maxpooling (with tensorflow).



Epoch by epoch (epoch 40)



True segmentation: alternating colors. Predicted segmentation: dashed lines.

Labels are transformed into constraints. Intuitively, the problem is:

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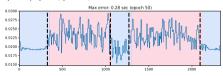
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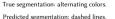
This setting can be used to learn a deep representation.

Here, two layers of temporal separable convolutions and maxpooling (with tensorflow).



Epoch by epoch (epoch 50)





Labels are transformed into constraints. Intuitively, the problem is:

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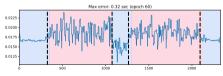
Two samples are *dissimilar* if they belong to consecutive regimes.

This setting can be used to learn a deep representation.

Here, two layers of temporal separable convolutions and maxpooling (with tensorflow).



Epoch by epoch (epoch 60)



True segmentation: alternating colors. Predicted segmentation: dashed lines.

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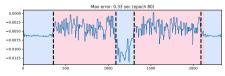
Two samples are *dissimilar* if they belong to consecutive regimes.

This setting can be used to learn a deep representation.

Here, two layers of temporal separable convolutions and maxpooling (with tensorflow).



Epoch by epoch (epoch 80)



True segmentation: alternating colors. Predicted segmentation: dashed lines.

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Two samples are *similar* if they belong to the same regime.

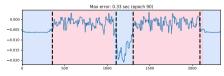
Two samples are *dissimilar* if they belong to consecutive regimes.

This setting can be used to learn a deep representation.

Here, two layers of temporal separable convolutions and maxpooling (with tensorflow).



Epoch by epoch (epoch 90)



True segmentation: alternating colors. Predicted segmentation: dashed lines.

Labels are transformed into constraints. Intuitively, the problem is:

• Learn a transformation Ψ such that $d(\Psi(x_t), \Psi(x_s)) \leq u$ if x_t and x_s similar $d(\Psi(x_t), \Psi(x_s)) \geq l$ if x_t and x_s dissimilar

(u > 0 and l > 0)

Two samples are *similar* if they belong to the same regime.

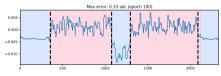
Two samples are *dissimilar* if they belong to consecutive regimes.

This setting can be used to learn a deep representation.

Here, two layers of temporal separable convolutions and maxpooling (with tensorflow).



Epoch by epoch (epoch 100)



True segmentation: alternating colors. Predicted segmentation: dashed lines.

Conclusion

- Code for those experiments will be available on my GitHub github.com/deepcharles.
- New methods are frequently implemented in ruptures.
- Extensions to graph/network data soon.
- > Differentiable dynamic programming for end-to-end unsupervised representation learning.

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